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# On the equity-efficiency trade off in aggregating infinite utility streams ☆

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**Abstract**

We compare two different approaches to aggregating infinite utility streams, following some principles of equity and efficiency. We find that the consequentialist equity axiom, Hammond Equity for the Future, is more demanding than its procedural equity counterpart, Anonymity.


*Keywords:* Weak Dominance axiom; Strong Pareto axiom; Hammond Equity for the Future; Social welfare function

*JEL classification:* D60; D70; D90

## 1. Introduction

In resolving the conflict between infinite generations formalized by ranking infinite utility streams satisfying some form of equity and efficiency (we call this the problem of intertemporal social choice) two approaches have been followed. The first approach following Diamond (1965) (we will call this the Diamond approach) seeks the existence of *continuous* welfare orders satisfying the equity and efficiency axioms. The second approach, due to Basu and Mitra (2003) (we will call this the Basu–Mitra approach),

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studies the existence of a *social welfare function* (SWF) satisfying the equity and efficiency axioms. No demand on continuity of the SWF is made in the latter approach.

Two classes of equity axioms have generated sufficient interest in the literature. The first is a *procedural* equity axiom that demands the use of only utility information in the social evaluation of infinite utility streams. In particular identities of generations have no role to play in the evaluation. The axiom of interest is Anonymity[1] (e.g., Diamond, 1965; Svensson, 1980; Basu and Mitra, 2003). The second class of equity axioms that have been discussed is called consequentialist equity (Suzumura and Shinotsuka, 2003; Hara et al., 2005; Asheim and Tungodden, 2004; Asheim et al., 2005). The equity axioms in this class express a preference for a more egalitarian distribution of utilities. In this paper we focus on an appealing consequentialist equity axiom, called "Hammond Equity for the Future", introduced by Asheim and Tungodden (2004) and analyzed by Asheim et al. (2005). This axiom expresses a weak preference for profiles where the sacrifice of the present generation makes all future generations better off by a constant utility amount.

This paper provides an answer to the following question: (1) is the possibility of resolving the problem of intertemporal social choice sensitive to the "approach" in which the problem is addressed? Loosely speaking, our purpose is to compare the "technical requirements" of continuity and representability in the context of evaluating infinite utility streams.

For a meaningful comparison between the approaches and the two equity axioms, we fix the domain of "utility streams" as a countable Cartesian product of the closed interval [0,1] and also a particular form of the efficiency postulate.

To that effect we observe that both Diamond's (1965, p. 176) impossibility theorem[2] holds for a weaker axiom of efficiency (Weak Dominance (WD)).[3] Asheim et al.'s (2005) Proposition 2 shows that it is impossible to aggregate infinite utility streams in a continuous binary relation that satisfy a restricted sensitivity axiom[4] (weaker than Weak Dominance) and a restricted upper semi-continuity axiom (weaker than the continuity axiom used in Diamond (1965)). The conclusion of Proposition 2 in Asheim et al. (2005) implies that the impossibility also holds when the restricted sensitivity is replaced by the stronger Weak Dominance axiom. In tracing the boundary of what is possible using the SWF approach to the problem of intertemporal social choice, Basu and Mitra (in press) report that it is possible to combine Anonymity and Weak Dominance in a SWF.

Table 1 summarizes the state of the art with special emphasis to the two equity axioms, Anonymity and Hammond Equity for the Future (HEF).

From the table it is evident that under Anonymity the two approaches give different answers to the intertemporal social choice problem; that is, continuity is more demanding than representability in this context.

To compare the two approaches under HEF we need to answer the following question: does there exist a SWF satisfying Hammond Equity for the Future and Weak Dominance? We find that there is no

[1] Anonymity states that if the utility stream $y$ is a finite permutation of $x$ then $x$ and $y$ should be declared as socially indifferent.

[2] The proof of this theorem is attributed to M. Yaari.

[3] The axiom used in Diamond (1965) is Strong Pareto, which can be stated as follows: If in a utility stream $x$ there is at least one generation which is strictly better off than the corresponding generation in profile $y$, with the other generations being no worse off in $x$ than in $y$, then society should strictly prefer $x$ over $y$.

[4] The restricted sensitivity axiom can be stated as follows: There exists real numbers $u; v$ with $u > v$ such that the infinite utility stream with $u$ as the first period utility and utility $v$ for all other generations is strictly preferred to the utility stream with the constant utility $v$.

Table 1
Summary of findings in the two different approaches

| | Diamond approach | Basu–mitra Approach |
|---|---|---|
| Anonymity and WD | Impossibility | Possibility |
| HEF and WD | Impossibility | |

social welfare function which simultaneously satisfies HEF and WD. This shows that with HEF as an equity axiom we can only aggregate infinite utility streams with a continuous welfare function if we use a weaker efficiency axiom (restricted weak Pareto; see Asheim et al., 2005). On the other hand if one is willing to give up continuity, then welfare orders satisfying HEF and WD can be shown to exist. Thus, continuity and representability are equally demanding in this context. From the point of aggregating infinite utility streams equitably, HEF seems to be a more demanding equity axiom than Anonymity, since *both* approaches to the intertemporal social choice problem using HEF give a negative result when the efficiency axiom used is Weak Dominance. We stress that this apparently negative result can be avoided by weakening either the efficiency postulate or the continuity requirement. So a compromise that generates a possibility of ranking infinite utility streams is open to debate and *does not* necessarily call for abandoning the appealing equity postulate, Hammond Equity for the Future.

The paper is organized as follows. Mathematical preliminaries and axioms are introduced in Section 2 and the main result is reported in Section 3.

## 2. Preliminaries

### 2.1. Notation and definitions

Let $\mathbb{N}$ denote, as usual, the set of natural numbers $\{1, 2, 3, \ldots\}$. Let $Y$ denote the closed interval [0,1], and let $X$ denote the set $Y^{\mathbb{N}}$. We let $X$ be the domain of utility sequences (also, referred to as "utility streams"). So we write $x \equiv (x_1, x_2, \ldots) \in X$ if and only if $x_n \in [0,1]$ for all $n \in \mathbb{N}$. A constant sequence satisfies $x_n = u$ for all $n \in \mathbb{N}$ for some $u \in [0,1]$, and is written as $(u)_{\text{con}}$.

For $x \in X$, and $N \in \mathbb{N}$, we denote $(x_1, \ldots, x_N)$ by ${}_1x_N$ and $(x_{N+1}, x_{N+2}, \ldots)$ by ${}_{N+1}x$. So given any $x \in X$ and $N \in \mathbb{N}$, we can write $x = ({}_1x_N, {}_{N+1}x)$. If $x, y \in X$, and $N \in \mathbb{N}$, we write $z = ({}_1x_N, y)$ to denote the element $z \in X$, satisfying $z_k = x_k$ for all $k \in \{1, \ldots, N\}$ and $z_{N+k} = y_k$ for all $k \in \mathbb{N}$.

For $y,z \in \mathbb{R}^{\mathbb{N}}$, we write $y \geq z$ if $y_i \geq z_i$ for all $i \in \mathbb{N}$; and, we write $y > z$ if $y \geq z$, and $y \neq z$.

A *social welfare function* (SWF) is a function, $W$, from $X$ to $\mathbb{R}$:

### 2.2. Axioms

We will be concerned with two axioms on the social welfare function, $W$. The equity axiom states that if the sacrifice by the present generation makes the constant utility level of all future generations higher, society should accept this trade off as being weakly welfare enhancing.

**Axiom 1** (*Hammond Equity for the Future*). If $x, y \in X$, such that $x_1 > y_1 > u > v$, $x = (x_1, (v)_{\text{con}})$ and $y = (y_1, (u)_{\text{con}})$ then, $W(y) \geq W(x)$.

The efficiency axiom, Weak Dominance, can be stated formally as follows.

**Axiom 2** (*Weak Dominance*). If $x$, $y \in X$ and there is some $j \in \mathbb{N}$ such that $y_j > x_j$ and for all $t \neq j$ $x_t = y_t$ then, $W(y) > W(x)$.

We do not want to endorse or justify the use of this axiom, which is clearly weaker than the Strong Pareto axiom. Its purpose is to present a result such that the two different approaches (Diamond and Basu–Mitra) to the problem of intertemporal social choice are comparable as is illustrated in Table 1.

## 3. An impossibility result

In this section we address the question of existence of welfare functions satisfying Hammond Equity for the Future and Weak Dominance. The method of proof is similar to the one used in Basu and Mitra (2003, Theorem 1).

**Theorem 1.** *There is no SWF satisfying the axioms of Hammond Equity for the Future and Weak Dominance*.

**Proof.** Suppose, on the contrary, there exists a SWF $W : X \rightarrow \mathbb{R}$, satisfying the axioms of Hammond Equity for the Future and Weak Dominance. Let us denote by $Z$ the open interval (1/8, 1/4). For any $p \in Z$, let us define

$$x(p) = \left(2p, (p/2)_{\text{con}}\right)$$

$$y(p) = \left(p, (p/2)_{\text{con}}\right).$$

Clearly $x(p)$, $y(p) \in X$, and we have:

$$W(x(p)) > W(y(p)) \tag{1}$$

by the Weak Dominance axiom. For each $p \in Z$ denote by $I(p)$ the non-degenerate interval $(W(y(p)), W(x(p)))$.

For any $p' < p$, we will show that $W(x(p')) \leq W(y(p))$. The choice of $Z$ implies $2p' > p$ for every $p' < p$ and $p', p \in Z$. From the definitions of $x(p')$ and $y(p)$ it is clear that $x(p')_1 > y(p)_1 > y(p)_n > x(p')_n$ for all $n \geq 2$. This implies $W(x(p')) \leq W(y(p))$ by the Hammond Equity for the Future axiom.

Thus we can associate with each $p \in Z$ a non-degenerate interval $I(p)$ of the real line. Moreover for any $p' \neq p$ with $p', p \in Z$, we must have $I(p) \cap I(p') = \varnothing$; that is, for distinct reals in $Z$ the associated intervals must be non-overlapping. This means that to each real number in $Z$ (an uncountable set) we can associate a distinct rational (from the interval $I(p)$) contradicting the countability of the set of rationals. □

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